

Phenomenology of Causal Dynamical Triangulations

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The four dimensional Causal Dynamical Triangulations (CDT) approach to quantum gravity is already more than ten years old theory with numerous unprecedented predictions such as non-trivial phase structure of gravitational field and dimensional running. Here, we discuss possible empirical consequences of CDT derived based on the two features of the approach mentioned above. A possibility of using both astrophysical and cosmological observations to test CDT is discussed. We show that scenarios which can be ruled out at the empirical level exist.

Keywords: Causal Dynamical Triangulations; Phenomenology of quantum gravity; Cosmology.

1. Introduction

The characteristic feature of physical systems composed of a huge number of non-linearly coupled degrees of freedom is emergence of the so-called *phases*, which correspond to different forms of internal organization.

The gravitational field seem to fulfill the criteria for the non-trivial phase structure to occur. This is because the system is described by non-linear field theory characterized by infinite number of degrees of freedom^a.

This presumption is materialized in the results obtained within Causal Dynamical Triangulations (CDT)¹ approach, which aims to describe quantum nature of the gravitational interactions by employing path integral formulation of quantum mechanics. The most up-to-date studies of CDT predict existence of three phases of gravity, together with an additional *bifurcation* sub-phase². The phases are separated by transition lines, among which first and second order phase transitions have already been detected³.

The quantity which is especially handy and useful in characterizing phases of gravity is the spectral dimension $d_S(\sigma)$, employing a random walk process on the considered quantum space-time. In particular, in the “most classical” phase *C*, the spectral dimension takes the value 4 for large diffusion time σ . However, at the short scales (small diffusion times) the value of d_S decreases, which can be captured by the following parametrization:

$$d_S(\sigma) = 4 - \frac{2 - \epsilon}{1 + \sigma E_*^2}. \quad (1)$$

Here, E_* is a characteristic energy scale of the dimensional reduction and the value

^aIn quantum version of the theory, the number of degrees of freedom theory is expected to be huge but finite.

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of ϵ depends on the location in the phase C at the phase diagram. The values of ϵ provided by the numerical simulations range from $\epsilon \approx 0$ ($d_S(0) \approx 2$)⁴ to $\epsilon \approx -1/2$ ($d_S(0) \approx 3/2$)⁵.

2. Modified dispersion relation

The definition of spectral dimension is rooted in the diffusion process, which depends on spectra of Laplace operator defined on a given quantum manifold. As discussed in Ref. 6, (under certain assumptions) form of the Laplace operator and consequently a dispersion relation for massless particles can be reconstructed from the diffusion time dependence of the spectral dimension. In particular, assuming the dispersion relation in the form $E = \Omega(p)$, the following asymptotic behaviors of the $\Omega(p)$ function are obtained with use of Eq. (1):

$$\Omega_{\text{IR}}(p) \approx p + \frac{E_*}{15}(2 - \epsilon) \left(\frac{p}{E_*} \right)^3 \quad \text{and} \quad \Omega_{\text{UV}}(p) \approx \frac{2}{3} E_* \left(\frac{p}{E_*} \right)^{3-3\epsilon}. \quad (2)$$

The IR approximation can be applied to study propagation of high energy astrophysical photons. For example, using observational constraints on the energy-dependence of the group velocity ($v_{\text{gr}} := \frac{\partial \Omega(p)}{\partial p}$) of photons from the GRB 090510 source⁷ one can derive the following constraint on the energy scale of the dimensional reduction: $E_* > 6.7 \cdot 10^{10} \text{ GeV}$ at (95%CL). The “low-energy” (below around 10 EeV) dimensional reduction is, therefore, observationally excluded.

Another possible application of the modified dispersion relation $E = \Omega(p)$ are cosmological perturbations in the early universe. At the phenomenological level, the dispersion relation can be introduced by replacing the momentum-space Laplace operator Δ_k contributing to the Hamiltonian of the type (details depend on which kind of the cosmological perturbations is considered):

$$H_\phi = \frac{1}{2} \int d^3 \mathbf{k} \left\{ \frac{1}{a^2} \pi_{\mathbf{k}} \pi_{-\mathbf{k}} - \phi_{\mathbf{k}} \underbrace{[-a^2 \Omega(k/a)^2]}_{=\Delta_k} \phi_{-\mathbf{k}} \right\}, \quad (3)$$

such that $\Delta_k \rightarrow -k^2$ in the classical limit and a denotes a scale factor.

It is worth noticing that the method of introducing the effects of dimensional reduction applied here differs from the one considered in Ref. 8. In that reference, modified dispersion relation has been introduced at the level of time-dependent speed of propagation. In our opinion, it is better justified to introduce the effect of modified dispersion relation at the level of the Fourier space representation of the Hamiltonian where the dispersion relation contributes explicitly.

Analysis of the vacuum-normalized perturbations described by the Hamiltonian of the type (3) leads to the following expression for the spectral index of the scalar perturbations:

$$n_S - 1 \equiv \frac{d \ln \mathcal{P}(k = k_H)}{d \ln k} \approx \frac{3\epsilon r}{r + 48(\epsilon - 1)}. \quad (4)$$

Here, $\mathcal{P}(k = k_H)$ is the amplitude of the scalar perturbations at the Hubble radius crossing (see Fig. 1) and r is the tensor-to-scalar ratio of the primordial perturbations.

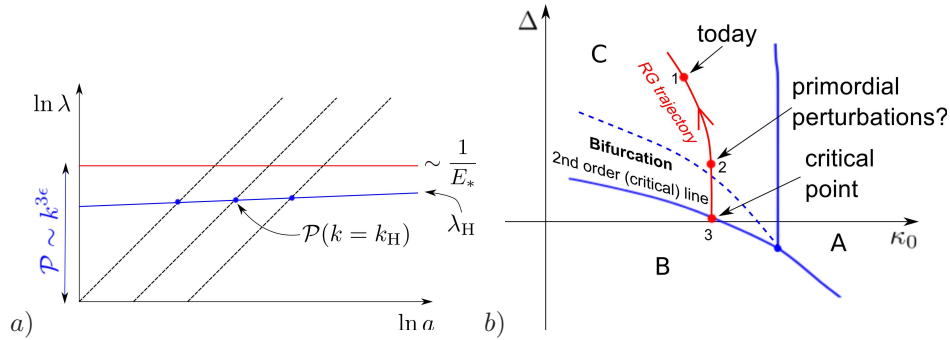


Fig. 1. a) Red-shifting of physical length scales, Hubble radius and scale of the dimensional reduction. b) Phase diagram of CDT and speculated physical RG trajectory.

During derivation of Eq. (4) it has been assumed that the Hubble radius is smaller than the length scale of the dimensional reduction. As discussed in Ref. 6, this leads to predictions being in conflict with the up-to-date PLANCK and BICEP II data. If the case when the Hubble radius is much bigger than the scale of the dimensional reduction the classical results which (for certain type of inflationary potential) agree with the cosmological data are recovered.

3. Phase transitions and gravitational defects

So far we considered points in the phase C corresponding either to current state of the universe in case of the astrophysical constraints (point 1 in Fig. 1) or to the state of the universe when the primordial cosmological perturbations were formed (point 2 in Fig. 1). The two points are characterized by different energy scales (energy densities) and are connected by Renormalization Group (RG) trajectory realized in the observed Universe.

A worth considering possibility is that following backward the RG trajectory one ends up at the second order transition line (point 3 in Fig. 1), which separates the bifurcation sub-phase of the phase C from the non-geometric phase B . The phase transition, being an example of the *geometrogenesis* process, provides numerous prospects for building phenomenology of CDT.

In particular, introduction of time scale to the $B - C$ phase transition lead us to the domain of non-equilibrium processes. If one would pass across the transition line infinitely slowly the system would have enough time to relax to a single global new ground state. However, passing through the transition point in a finite amount of time does not allow to relax to a single ground state and collection of the so-called

domains is formed (by virtue of the Kibble-Zurek mechanism).

Some properties of the non-equilibrium transition might be estimated with use of the characteristics of the equilibrium phase transitions. In particular, typical sizes of the domains are $\xi \approx \xi_0 \left(\frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{\nu z + 1}} \approx l_{\text{Pl}}$, where l_{Pl} is the Planck length. In the case without cosmic inflation, the present size of the domains might be estimated as follows: $\xi_{\text{today}} \approx \xi \frac{T_{\text{Pl}}}{T_{\text{CMB}}} \sim 1 \text{ mm}$.

At the boundaries separating the different ground states (domains) the gravitational defects are formed. With use of ξ_{today} an average defect concentration is $d \sim \frac{1}{\xi_{\text{today}}^3} \sim \frac{1}{\text{mm}^3}$. Because any of the defects is observed, the gravitational version of the topological defect problem must be solved. The solution might be provided by the phase of inflation, diluting the concentration of defects to the level beyond the observational threshold. On the other hand (for obvious reasons) presence of the cosmic inflation makes confrontation of the gravitational phase transitions with observations much more difficult.

4. Conclusions

We have presented some possible paths allowing for construction of phenomenology of the CDT approach to quantum gravity. While at the moment none of the predictions of the CDT can be approved, some scenarios seem to be in conflict with the up-to-date observational data. The results encourage to take further attempts in Socratic debate with Nature on a role of CDT in description of gravitational phenomena at the Planck scale.

Acknowledgments

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